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A critical investigation into the heat and mass transfer analysis of counterflow wet-cooling towers

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Abstract

This study gives a detailed derivation of the heat and mass transfer equations of evaporative cooling in wet-cooling towers. The governing equations of the rigorous Poppe method of analysis are derived from first principles. The method of Poppe is well suited for the analysis of hybrid cooling towers as the state of the outlet air is accurately predicted. The governing equations of the Merkel method of analysis are subsequently derived after some simplifying assumptions are made. The equations of the effectiveness-NTU method applied to wet-cooling towers are also presented. The governing equations of the Poppe method are extended to give a more detailed representation of the Merkel number. The differences in the heat and mass transfer analyses and solution techniques of the Merkel and Poppe methods are described with the aid of enthalpy diagrams and psychrometric charts. The psychrometric chart is extended to accommodate air in the supersaturated state.

1. Introduction

The governing equations for heat and mass transfer in the fill of a counterflow cooling tower are derived in this paper. The governing equations for the Merkel [1], Poppe and Rögener [2] and e-NTU [3] methods are presented. The Merkel method, developed in the 1920s, relies on several critical assumptions to reduce the solution to a simple hand calculation. Because of these assumptions, however, the Merkel method does not accurately represent the physics of the heat and mass transfer process in the cooling tower fill.

The critical simplifying assumptions of the Merkel method are [4]:

- The Lewis factor relating heat and mass transfer is equal to 1. This assumption has a small influence but affects results at low ambient temperatures.
- The air exiting the tower is saturated with water vapor and it is characterized only by its enthalpy. This assumption regarding saturation has a negligible influence above an ambient temperature of 20 °C but is of importance at lower temperatures.
- The reduction of water flow rate by evaporation is neglected in the energy balance. This energy balance simplification has a greater influence at elevated ambient temperatures.

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Nomenclature

A	area, m^2	*Subscripts*	
a	surface area per unit volume, m^{-1}	a	air
C	heat capacity rate mc_p, W/K, or fluid capacity rate ratio , C_{min}/C_{max},	c	convection heat transfer, or cold
c_p	specific heat at constant pressure, J/kg K	e	*e-NTU* method
e	effectiveness	fi	fill
G	mass velocity, kg/m^2s	h	hot
h	heat transfer coefficient, W/m^2K	i	inlet
h_d	mass transfer coefficient, kg/m^2s	M	Merkel method
i	enthalpy, J/kg	m	mean, or mass transfer
i_{fg}	latent heat, J/kg	max	maximum
Le_f	Lewis factor, $h/(c_p h_d)$	min	minimum
m	mass flow rate, kg/s	o	outlet
Me	Merkel number, $h_d a_{fi} L_{fi}/G_w$	P	Poppe method
NTU	number of transfer units	s	saturation
p	pressure, N/m^2, or Pa	ss	supersaturated
Q	heat transfer rate, W	v	vapor
T	temperature, °C or K	w	water
U	overall heat transfer coefficient, W/m^2K	wb	wetbulb
w	humidity ratio, kg water vapor/kg dry air		
z	coordinate, or elevation, m		

Bourillot [4] stated that the Merkel method is simple to use and can correctly predict cold water temperature when an appropriate value of the coefficient of evaporation is used. In contrast, it is insufficient for the estimation of the characteristics of the warm air leaving the fill and for the calculation of changes in the water flow rate due to evaporation. These quantities are important to estimate water consumption and to predict the behavior of plumes exiting the cooling tower.

Jaber and Webb [3] developed the equations necessary to apply the *e-NTU* method directly to counterflow or crossflow cooling towers. This approach is particularly useful in the latter case and simplifies the method of solution when compared to a more conventional numerical procedure. The *e-NTU* method is based on the same simplifying assumptions as the Merkel method.

The method of Poppe, developed in the 1970s, does not make the simplifying assumptions of Merkel. Predictions from the Poppe formulation result in values of evaporated water flow rate that are in good agreement with full scale cooling tower test results. In addition, the Poppe method predicts the water content of the exiting air accurately [4,5]. The fact that the Poppe method predicts the water content of the exiting air accurately is a very important consideration in the design of hybrid cooling towers [6].

Section 2 gives the detailed derivation of the governing equations according to the Poppe method. The Merkel method is discussed in Section 3. Section 3 also shows where in the derivation of the governing equations, according to the Poppe method, the Merkel method assumptions take effect to simplify the solution of the equations considerably. The *e-NTU* method, which is based on the same simplifying assumptions as the Merkel method, is presented in Section 4. The discussion on the differences between the methods and the conclusion are presented in Sections 5 and 6 respectively.

2. Poppe method

The following two subsections where the governing equations of the evaporative cooling process are derived according to the Poppe method of analysis, are adapted from Bourillot [5], Poppe and Rögener [2], Kröger [7] and Baard [8]. The procedure to calculate the Merkel number, according to the Poppe method, is extended in the current derivation to give a more detailed representation of the integration of the Merkel number in the counterflow transfer region. The governing equations that follow can be solved by the fourth order Runge–Kutta method. Refer to Appendix B for the procedure to solve the governing equations by the Runge–Kutta method.

2.1. Governing equations for heat and mass transfer in fill for unsaturated air

Fig. 1 shows a control volume in the fill of a counterflow wet-cooling tower. Fig. 2 shows an airside control volume of the fill illustrated in Fig. 1.

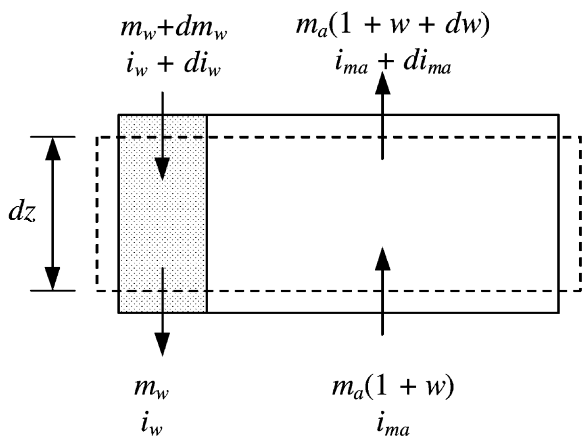

Fig. 1. Control volume of counterflow fill.

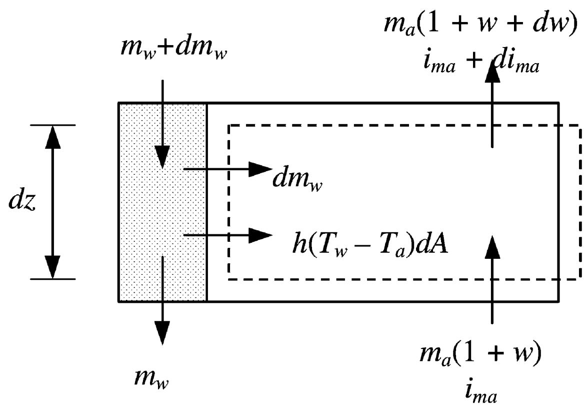

Fig. 2. Air side control volume of the fill.

A mass balance for the control volume in Fig. 1 yields

$$\mathrm{d}m_{\mathrm{w}} = m_{\mathrm{a}}\mathrm{d}w \tag{1}$$

The energy balance for the control volume of the fill in Fig. 1 is as follows:

$$m_{\mathrm{a}}\mathrm{d}i_{\mathrm{ma}} - m_{\mathrm{w}}\mathrm{d}i_{\mathrm{w}} - i_{\mathrm{w}}\mathrm{d}m_{\mathrm{w}} = 0 \tag{2}$$

where i_{ma} is the enthalpy of the air–vapor mixture expressed by Eq. (A.1).

Substitute Eq. (1) into Eq. (2) to find upon rearrangement,

$$\mathrm{d}T_{\mathrm{w}} = \frac{m_{\mathrm{a}}}{m_{\mathrm{w}}}\left(\frac{1}{c_{\mathrm{pw}}}\mathrm{d}i_{\mathrm{ma}} - T_{\mathrm{w}}\mathrm{d}w\right) \tag{3}$$

Consider the interface between the water and the air in Fig. 2. An energy balance at the interface yields

$$\mathrm{d}Q = \mathrm{d}Q_{\mathrm{m}} + \mathrm{d}Q_{\mathrm{c}} \tag{4}$$

where $\mathrm{d}Q_{\mathrm{m}}$ is the enthalpy transfer due to difference in vapor concentration between the saturated air at the interface and the mean stream air and $\mathrm{d}Q_{\mathrm{c}}$ is the sensible heat transfer due to the difference in temperature. The mass transfer at the interface is expressed by

$$\mathrm{d}m_{\mathrm{w}} = h_{\mathrm{d}}(w_{\mathrm{sw}} - w)\mathrm{d}A \tag{5}$$

The corresponding enthalpy transfer for the mass transfer in Eq. (5) is

$$\mathrm{d}Q_{\mathrm{m}} = i_{\mathrm{v}}\mathrm{d}m_{\mathrm{w}} = i_{\mathrm{v}}h_{\mathrm{d}}(w_{\mathrm{sw}} - w)\mathrm{d}A \tag{6}$$

The enthalpy of the water vapor, i_{v}, at the bulk water temperature, T_{w}, is given by

$$i_{\mathrm{v}} = i_{\mathrm{fgwo}} + c_{\mathrm{pv}}T_{\mathrm{w}} \tag{7}$$

The convective heat transfer from Fig. 2 is given by

$$\mathrm{d}Q_{\mathrm{c}} = h(T_{\mathrm{w}} - T_{\mathrm{a}})\mathrm{d}A \tag{8}$$

The temperature differential in Eq. (8) can be replaced by an enthalpy differential. The enthalpy of saturated air evaluated at the local bulk water temperature is given by

$$i_{\mathrm{masw}} = c_{\mathrm{pa}}T_{\mathrm{w}} + w_{\mathrm{sw}}(i_{\mathrm{fgwo}} + c_{\mathrm{pv}}T_{\mathrm{w}}) \tag{9}$$

Substitute Eq. (7) into Eq. (9), rearrange and find

$$i_{\mathrm{masw}} = c_{\mathrm{pa}}T_{\mathrm{w}} + wi_{\mathrm{v}} + (w_{\mathrm{sw}} - w)i_{\mathrm{v}} \tag{10}$$

The enthalpy of the air–water vapor mixture per unit mass of dry air which, according to Eq. (A.1), is expressed by

$$i_{\mathrm{ma}} = c_{\mathrm{pa}}T_{\mathrm{a}} + w(i_{\mathrm{fgwo}} + c_{\mathrm{pv}}T_{\mathrm{a}}) \tag{11}$$

Subtract Eq. (11) from (10). The resultant equation can be simplified if the small differences in specific heats, which are evaluated at different temperatures, are ignored.

$$T_{\mathrm{w}} - T_{\mathrm{a}} = \frac{(i_{\mathrm{masw}} - i_{\mathrm{ma}}) - (w_{\mathrm{sw}} - w)i_{\mathrm{v}}}{c_{\mathrm{pma}}} \tag{12}$$

where c_{pma} is given by Eq. (A.5).

Substitute Eq. (12) into Eq. (8). Substitute the resultant equation and Eq. (6) into Eq. (4) to find after rearrangement,

$$\mathrm{d}Q = h_{\mathrm{d}}\left[\frac{h}{c_{\mathrm{pma}}h_{\mathrm{d}}}(i_{\mathrm{masw}} - i_{\mathrm{ma}}) + \left(1 - \frac{h}{c_{\mathrm{pma}}h_{\mathrm{d}}}\right)i_{\mathrm{v}}(w_{\mathrm{sw}} - w)\right]\mathrm{d}A \tag{13}$$

$h/c_{\mathrm{pma}}h_{\mathrm{d}}$ in Eq. (13) is known as the Lewis factor Le_{f} and is an indication of the relative rates of heat and mass transfer in an evaporative process. Bosnjakovic [9] developed an empirical relation for the Lewis factor Le_f for air–water vapor systems. The Lewis factor for unsaturated air, according to Bosnjakovic [9] is given by

$$Le_f = 0.865^{0.667}\left(\frac{w_{sw}+0.622}{w+0.622}-1\right)\Big/\left[\ln\left(\frac{w_{sw}+0.622}{w+0.622}\right)\right] \tag{14}$$

Refer to Kloppers [10] for a discussion on the derivation and development of Eq. (14). Other equations, given by Kloppers [10], can be employed to express the Lewis factor. He shows that it is very important to employ the same equation or definition for the Lewis factor in the fill performance analysis and in the subsequent cooling tower performance analysis if the water outlet temperature is to be calculated accurately. The water evaporation rate, however, is a function of the actual value of the Lewis factor, especially when the ambient air is relatively warm.

The enthalpy transfer to the air stream from Eq. (13) is

$$\begin{aligned} di_{ma} &= \frac{1}{m_a}dQ \\ &= \frac{h_d dA}{m_a}[Le_f(i_{masw}-i_{ma})+(1-Le_f)i_v(w_{sw}-w)] \end{aligned} \tag{15}$$

For a one-dimensional model of the cooling tower fill, where the available area for heat and mass transfer is the same at any horizontal section through the fill, the transfer area for a section dz is usually expressed as

$$dA = a_{fi}A_{fr}dz \tag{16}$$

where a_{fi} is the area density of the fill, i.e. the wetted area divided by the corresponding volume of the fill and A_{fr} is the corresponding frontal area or face area. Substitute Eq. (16) into Eq. (15) and find

$$\frac{di_{ma}}{dz} = \frac{h_d a_{fi}A_{fr}}{m_a}[Le_f(i_{masw}-i_{ma})+(1-Le_f)i_v(w_{sw}-w)] \tag{17}$$

Substitute Eqs. (5) and (15) into Eq. (2), rearrange and find,

$$\begin{aligned} m_w di_w =& h_d dA[i_{masw}-i_{ma}+(Le_f-1)[i_{masw}-i_{ma} \\ &-(w_{sw}-w)i_v]-(w_{sw}-w)c_{pw}T_w] \end{aligned} \tag{18}$$

Find upon rearrangement of Eq. (3)

$$\frac{dw}{dT_w} = \frac{1}{c_{pw}T_w}\frac{di_{ma}}{dT_w}-\frac{1}{T_w}\frac{m_w}{m_a} \quad \text{or}$$

$$\frac{dw}{dT_w} = \frac{di_{ma}}{T_w di_w}-\frac{1}{T_w}\frac{m_w}{m_a} \tag{19}$$

Substitute Eqs. (15) and (18) into Eq. (19) and find,

$$\frac{dw}{dT_w} = \frac{c_{pw}\frac{m_w}{m_a}(w_{sw}-w)}{i_{masw}-i_{ma}+(Le_f-1)[i_{masw}-i_{ma}-(w_{sw}-w)i_v]-(w_{sw}-w)c_{pw}T_w} \tag{20}$$

Substitute Eq. (20) into Eq. (19) and find,

$$\frac{di_{ma}}{dT_w} = \frac{m_w c_{pw}}{m_a}\left(1+\frac{(w_{sw}-w)c_{pw}T_w}{i_{masw}-i_{ma}+(Le_f-1)[i_{masw}-i_{ma}-(w_{sw}-w)i_v]-(w_{sw}-w)c_{pw}T_w}\right) \tag{21}$$

From Eqs. (1) and (5) find

$$h_d dA = \frac{m_a dw}{w_{sw}-w} \tag{22}$$

Divide both sides by m_w and introduce dT_w/dT_w to the right hand side of Eq. (22) and integrate to find

$$\int \frac{h_d}{m_w}dA = \int \frac{m_a}{m_w}\frac{dw/dT_w}{w_{sw}-w}dT_w \tag{23}$$

From Eq. (23) find

$$\frac{h_d A}{m_w} = \int \frac{m_a}{m_w}\frac{dw/dT_w}{w_{sw}-w}dT_w \tag{24}$$

Eq. (24) is defined as the Merkel number according to the Poppe method i.e.

$$Me_P = \int \frac{m_a}{m_w}\frac{dw/dT_w}{w_{sw}-w}dT_w \tag{25}$$

Upon substitution of Eq. (20) into Eq. (25) and differentiation of the latter with respect to the water temperature, find

$$\frac{Me_P}{dT_w} = \frac{c_{pw}}{i_{masw} - i_{ma} + (Le_f - 1)[i_{masw} - i_{ma} - (w_{sw} - w)i_v] - (w_{sw} - w)c_{pw}T_w} \tag{26}$$

The ratio of the mass flow rates m_w/m_a changes as the air moves towards the top of the fill. The change in the mass flow rate is determined by considering the control volume of a portion of the fill illustrated in Fig. 3.

The varying water mass flow rate can be determined from the known inlet water mass flow rate, m_{wi}. From the control volume in Fig. 3 a mass balance will yield,

$$m_{wi} = m_w + m_a(w_o - w) \tag{27}$$

After rearrangement of Eq. (27) find,

$$\frac{m_w}{m_a} = \frac{m_{wi}}{m_a}\left(1 - \frac{m_a}{m_{wi}}(w_o - w)\right) \tag{28}$$

From Eqs. (14), (20), (21) and (28) the air outlet conditions in terms of enthalpy and humidity ratio can be calculated. The value for w_o in Eq. (28) is not known a priori and the equations must therefore be solved by an iterative procedure.

The preceding system of equations is only applicable for unsaturated air. In some cases, the air can become saturated before it leaves the fill [7]. Because the water temperature is still higher than the temperature of the air, the potential for heat and mass transfer still exists. Under these conditions, the excess water vapor will condense as a mist.

2.2. Governing equations for heat and mass transfer in fill for supersaturated air

The control volumes in Figs. 1 and 2 are also applicable if the air is supersaturated. Since the excess water vapor will condense as a mist, the enthalpy of supersaturated air is expressed by

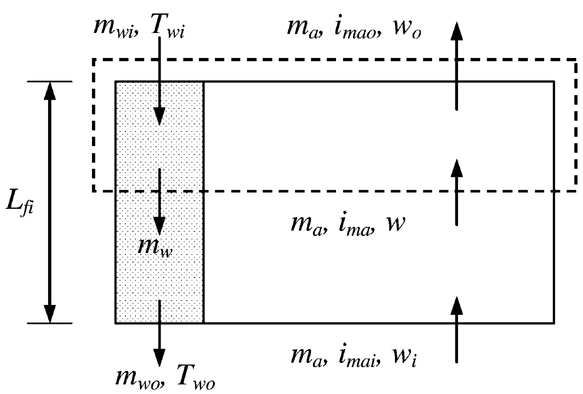

Fig. 3. Control volume of the fill.

$$i_{ss} = c_{pa}T_a + w_{sa}(i_{fgwo} + c_{pv}T_a) + (w - w_{sa})c_{pw}T_a \tag{29}$$

where w_{sa} is the humidity ratio of saturated air at temperature T_a.

Assume that the heat and mass transfer coefficients for supersaturated and unsaturated air are the same as proposed by Bourillot [5] and Poppe and Rögener [2]. The driving potential for mass transfer is the humidity ratio difference between the saturated air at the air–water interface and the saturated free stream air, thus

$$dm_w = h_d(w_{sw} - w_{sa})dA \tag{30}$$

The enthalpy driving potential for supersaturated air can be obtained by subtracting Eq. (29) from Eq. (10). By introducing,

$$(w - w_{sa})c_{pw}T_w - (w - w_{sa})c_{pw}T_w + w_{sa}c_{pv}T_w - w_{sa}c_{pv}T_w$$

which adds up to zero into the resultant enthalpy differential, the temperature differential can be obtained by manipulation

$$T_w - T_a = \frac{i_{masw} - i_{ss} - (w_{sw} - w_{sa})i_v + (w - w_{sa})c_{pw}T_w}{c_{pmas}} \tag{31}$$

where c_{pmas} is the specific heat of supersaturated air per unit mass and defined as

$$c_{pmas} = c_{pa} + w_{sa}c_{pv} + (w - w_{sa})c_{pw} \tag{32}$$

Proceeding along the same lines as in the case of unsaturated air, using Eqs. (30) and (31) instead of Eqs. (5) and (12) find for supersaturated air

$$di_{ma} = \frac{h_d dA}{m_a}[Le_f\{i_{masw} - i_{ss} - (w_{sw} - w_{sa})i_v + (w - w_{sa})c_{pw}T_w\} + (w_{sw} - w_{sa})i_v] \tag{33}$$

where the Lewis factor, Le_f, is equal to $h/h_d c_{pmas}$. Poppe employed the empirical relation of Bosnjakovic [9] to calculate the Lewis factor, which for supersaturated air is given by

$$Le_f = 0.865^{0.667}\left(\frac{w_{sw} + 0.622}{w_{sa} + 0.622} - 1\right) \Big/ \left[\ln\left(\frac{w_{sw} + 0.622}{w_{sa} + 0.622}\right)\right] \tag{34}$$

Substitute Eqs. (30) and (33) into Eq. (2) and find after rearrangement

$$m_w di_w = m_w c_{pw} dT_w = h_d dA \times \begin{bmatrix} Le_f\{i_{masw} - i_{ss} - (w_{sw} - w_{sa})i_v + (w - w_{sa})c_{pw}T_w\} \\ +(w_{sw} - w_{sa})i_v - (w_{sw} - w_{sa})c_{pw}T_w \end{bmatrix} \tag{35}$$

By introducing

$$[i_{\text{masw}} - i_{\text{ss}} - (w_{\text{sw}} - w_{\text{sa}})i_{\text{v}} + (w - w_{\text{sa}})c_{\text{pw}}T_{\text{w}}] - [i_{\text{masw}} - i_{\text{ss}} - (w_{\text{sw}} - w_{\text{sa}})i_{\text{v}} + (w - w_{\text{sa}})c_{\text{pw}}T_{\text{w}}]$$

into the main parenthesis on right hand side of Eq. (35) the following equation yields after rearrangement:

$$m_{\text{w}}\text{d}i_{\text{w}} = m_{\text{w}}c_{\text{pw}}\text{d}T_{\text{w}} = h_{\text{d}}\text{d}A \times \left[\begin{matrix} i_{\text{masw}} - i_{\text{ss}} + (Le_{\text{f}} - 1)\left(\begin{matrix} i_{\text{masw}} - i_{\text{ss}} - (w_{\text{sw}} - w_{\text{sa}})i_{\text{v}} \\ +(w - w_{\text{sa}})c_{\text{pw}}T_{\text{w}} \end{matrix}\right) \\ +(w - w_{\text{sw}})c_{\text{pw}}T_{\text{w}} \end{matrix}\right] \tag{36}$$

$$\frac{\text{d}Me_{\text{p}}}{\text{d}T_{\text{w}}} = \frac{c_{\text{pw}}}{i_{\text{masw}} - i_{\text{ss}} + (Le_{\text{f}} - 1)\left[\begin{matrix} i_{\text{masw}} - i_{\text{ss}} - (w_{\text{sw}} - w_{\text{sa}})i_{\text{v}} \\ +(w - w_{\text{sa}})c_{\text{pw}}T_{\text{w}} \end{matrix}\right] + (w - w_{\text{sw}})c_{\text{pw}}T_{\text{w}}} \tag{43}$$

Substitute Eq. (30) into Eq. (1) and find,

$$h_{\text{d}}\text{d}A = \frac{m_{\text{a}}\text{d}w}{(w_{\text{sw}} - w_{\text{sa}})} \tag{37}$$

Substitute Eq. (37) into Eq. (36) and find,

$$\frac{\text{d}w}{\text{d}T_{\text{w}}} = \frac{c_{\text{pw}}\frac{m_{\text{w}}}{m_{\text{a}}}(w_{\text{sw}} - w_{\text{sa}})}{i_{\text{masw}} - i_{\text{ss}} + (Le_{\text{f}} - 1)\left[\begin{matrix} i_{\text{masw}} - i_{\text{ss}} - (w_{\text{sw}} - w_{\text{sa}})i_{\text{v}} \\ +(w - w_{\text{sa}})c_{\text{pw}}T_{\text{w}} \end{matrix}\right] + (w - w_{\text{sw}})c_{\text{pw}}T_{\text{w}}} \tag{38}$$

Substitute Eq. (38) into Eq. (19) and find upon rearrangement,

$$\frac{\text{d}i_{\text{ma}}}{\text{d}T_{\text{w}}} = c_{\text{pw}}\frac{m_{\text{w}}}{m_{\text{a}}}\left(1 + \frac{c_{\text{pw}}T_{\text{w}}(w_{\text{sw}} - w_{\text{sa}})}{i_{\text{masw}} - i_{\text{ss}} + (Le_{\text{f}} - 1)\left[\begin{matrix} i_{\text{masw}} - i_{\text{ss}} - (w_{\text{sw}} - w_{\text{sa}})i_{\text{v}} \\ +(w - w_{\text{sa}})c_{\text{pw}}T_{\text{w}} \end{matrix}\right] + (w - w_{\text{sw}})c_{\text{pw}}T_{\text{w}}}\right) \tag{39}$$

From Eqs. (1) and (30) find

$$h_{\text{d}}\text{d}A = \frac{m_{\text{a}}d_{\text{w}}}{w_{\text{sw}} - w_{\text{sa}}} \tag{40}$$

Divide both sides of Eq. (40) by m_{w}, introduce $\text{d}T_{\text{w}}/\text{d}T_{\text{w}}$ to the right hand side of Eq. (40) and integrate to find

$$\int \frac{h_{\text{d}}}{m_{\text{w}}}\text{d}A = \int \frac{m_{\text{a}}}{m_{\text{w}}}\frac{\text{d}w/\text{d}T_{\text{w}}}{w_{\text{sw}} - w_{\text{sa}}}\text{d}T_{\text{w}} \tag{41}$$

Eq. (41) is defined as the Merkel number according to the Poppe method i.e.

$$Me_{\text{p}} = \frac{h_{\text{d}}A}{m_{\text{w}}} = \int \frac{m_{\text{a}}}{m_{\text{w}}}\frac{\text{d}w/\text{d}T_{\text{w}}}{w_{\text{sw}} - w_{\text{sa}}}\text{d}T_{\text{w}} \tag{42}$$

Upon substitution of Eq. (38) into Eq. (42) and differentiation of the latter with respect to water temperature, find

From Eqs. (28), (34), (38) and (39), the air outlet conditions in terms of enthalpy and humidity ratio can be calculated.

The fourth order Runge–Kutta method is employed to solve the system of equations. Refer to Appendix B for the implementation of the Runge–Kutta method to solve the governing equations of the Poppe method. Refer to Kloppers [10] for detailed sample calculations to solve the governing equations according to the Poppe method.

3. Merkel method

To simplify the analysis of an evaporative cooling process Merkel [1] assumed that the evaporative loss is negligible, i.e. $dw = 0$ from Eq. (3), and that the Lewis factor is equal to one. Eqs. (17) and (3) of the counterflow evaporative process simplify respectively to

$$\frac{di_{ma}}{dz} = \frac{h_d a_{fi} A_{fr}}{m_a}(i_{masw} - i_{ma}) \tag{44}$$

and by dividing Eq. (3) by dz on both sides of Eq. (3) to

$$\frac{dT_w}{dz} = \frac{m_a}{m_w}\frac{1}{c_{pw}}\frac{di_{ma}}{dz} \tag{45}$$

Eqs. (44) and (45) describe respectively the change in the enthalpy of the air–water vapor mixture and the change in water temperature as the air travel distance changes. Eqs. (44) and (45) can be combined to yield upon integration the Merkel equation

$$Me_M = \frac{h_d A}{m_w} = \frac{h_d a_{fi} A_{fr} L_{fi}}{m_w} = \frac{h_d a_{fi} L_{fi}}{G_w} = \int_{T_{wo}}^{T_{wi}} \frac{c_{pw} dT_w}{(i_{masw} - i_{ma})} \tag{46}$$

where Me_M is the Merkel number according to the Merkel method. In the literature the notation frequently used for the Merkel number is KaV/L where $K = h_d$, $a = a_{fi}$ and $L = m_w$. It is not possible to calculate the state of the air leaving the fill according to Eq. (46). Merkel assumed that the air leaving the fill is saturated with water vapor. This assumption enables the approximate air temperature leaving the fill to be calculated.

Eq. (46) is not self-sufficient so it does not lend itself to direct mathematical solution [11,12]. The usual procedure is to integrate it in conjunction with an energy balance expressed by

$$m_w c_{pwm} dT_w = m_a di_{ma} \tag{47}$$

Fig. 4 shows the enthalpy curves of the air in a counterflow wet-cooling tower. The fill test results, from which Fig. 4 is generated, are given in Kröger [7]. The i_{ma} curve i.e. the enthalpy of the air as it moves through the fill, shown in Fig. 4, is linear due to the linear nature of Eq. (47). The i_{masw} curve is the saturation curve of the air at the water interface temperature. The potential for heat and mass transfer at a particular water temperature is the difference between i_{masw} and i_{ma}. The Merkel number, Me_M, of Eq. (46), is a function of the area under the $1/(i_{masw} - i_{ma})$ curve as shown in Fig. 4.

The integral in Eq. (46) needs to be evaluated by numerical integration techniques. The British Standard [13] and the Cooling Tower Institute [14,15] recommend that the four-point Chebyshev integration technique be employed. A discussion of the Chebyshev integration technique can also be found in Oosthuizen [16] and Mohiuddin and Kant [17]. Kelly [18] states that the Chebyshev procedure lacks accuracy when the approach (i.e. the difference between the water outlet temperature and the air inlet wetbulb temperature) is small (down to 0.56 °C). Any integration technique can be employed to solve Eq. (46) but it is strongly recommended that the same integration technique be employed in the fill performance analysis and the subsequent cooling tower performance analysis. The four-point Chebyshev integration technique essentially uses four intervals for the determination of the integral. Li and Priddy [19] and Mills [20] use thirteen and seven intervals respectively for numerical integration to determine the change of water and air enthalpy through the fill for a cooling range of approximately 14 °C. Li and Priddy [19] effectively employ a Riemann sum [21] to determine the integral while Mills [20] employs the composite trapezoidal rule [22]. It was found by the authors that the Chebyshev procedure is generally very accurate when compared to the composite Simpson rule with 100 intervals which has an error of the fourth order [23].

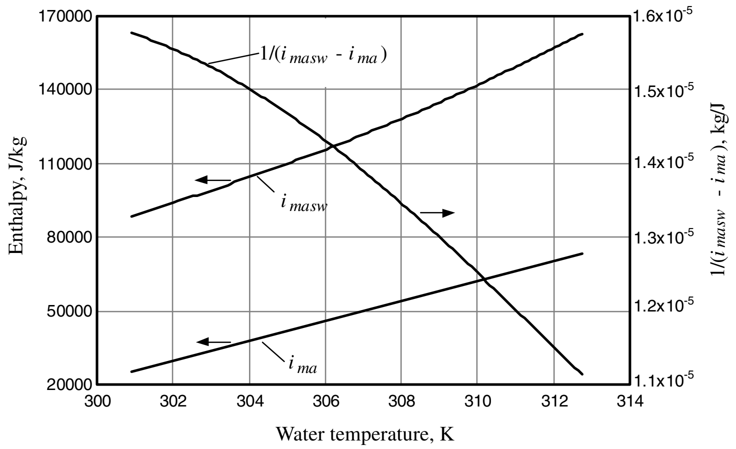

Fig. 4. Enthalpy diagram of the Merkel method.

As already mentioned, the driving potential in wet-cooling towers is the difference between the enthalpies i_{masw} and i_{ma} as shown in Fig. 4. The i_{ma} curve is obtained from Eq. (47) that ignores the change in water flow rate due to evaporation. The effect of evaporation on the energy balance is thus ignored for a second time. It was first ignored when Eq. (46) was derived. Baker and Shryock [11] investigated the effect of this second instance where the evaporation is ignored in the energy balance. They considered three different cases and found that the Merkel number increases with the more accurate representations of the energy balance. The Merkel number, however, does not increase as much for the most accurate case investigated as for the second most accurate case. The maximum increase in the Merkel number is 4.4%. Again, it is stressed that the same energy balance be employed in the fill performance analysis and the subsequent cooling tower performance analysis.

Curves are published in the literature to determine the Merkel number in Eq. (46) by graphical means from known air and water temperatures and air and water mass flow rates. Curves to determine the tower characteristic for counterflow towers, are given by the CTI [24]. Since the advent of high speed digital computers, these curves are less frequently used.

4. e-NTU method

Jaber and Webb [3] developed the equations necessary to apply the *e-NTU* method directly to counterflow or crossflow cooling towers. Kröger [7] gives a detailed derivation and implementation of the *e-NTU* method applied to evaporative air–water systems.

It can be shown according to Jaber and Webb [3] that

$$\frac{d(i_{masw}-i_{ma})}{(i_{masw}-i_{ma})} = h_d\left(\frac{di_{masw}/dT_w}{m_w c_{pw}} - \frac{1}{m_a}\right)dA \tag{48}$$

Eq. (48) corresponds to the heat exchanger *e-NTU* equation

$$\frac{d(T_h-T_c)}{(T_h-T_c)} = -U\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right)dA \tag{49}$$

Two possible cases of Eq. (48) can be considered where m_a is greater or less than $m_w c_{pw}/(di_{masw}/dT_w)$. The maximum of m_a and $m_w c_{pw}/(di_{masw}/dT_w)$ is denoted by C_{max} and the minimum by C_{min}. The gradient of the saturated air enthalpy–temperature curve is

$$\frac{di_{masw}}{dT_w} = \frac{i_{maswi}-i_{maswo}}{T_{wi}-T_{wo}} \tag{50}$$

The fluid capacity rate ratio is defined as

$$C = C_{min}/C_{max} \tag{51}$$

The effectiveness is given by

$$e = \frac{Q}{Q_{max}} = \frac{m_w c_{pw}(T_{wi}-T_{wo})}{C_{min}(i_{maswi}-\lambda-i_{mai})} \tag{52}$$

where λ is a correction factor, according to Berman [25], to improve the approximation of the i_{masw} versus T_w curve as a straight line. The correction factor, λ, is given by

$$\lambda = (i_{maswo}+i_{maswi}-2i_{maswm})/4 \tag{53}$$

where i_{maswm} denotes the enthalpy of saturated air at the mean water temperature. The number of transfer units for counterflow cooling towers is given by

$$\mathrm{NTU} = \frac{1}{1-C}\ln\frac{1-eC}{1-e} \tag{54}$$

If m_a is greater than $m_w c_{pw}/(di_{masw}/dT_w)$ the Merkel number according to the *e-NTU* method is given by

$$Me_e = \frac{c_{pw}}{di_{masw}/dT_w}\mathrm{NTU} \tag{55}$$

If m_a is less than $m_w c_{pw}/(di_{masw}/dT_w)$ the Merkel number according to the *e-NTU* method is given by

$$Me_e = \frac{m_a}{m_w}\mathrm{NTU} \tag{56}$$

5. Discussion

It is expected that the Poppe method will lead to more accurate results than those obtained by employing the Merkel and *e-NTU* methods, as it is the more rigorous method. The results of the Merkel and *e-NTU* methods of analysis are generally very close to each other as these methods are based on the same simplifying assumptions. The comparison between the Poppe and Merkel methods is shown on the psychrometric charts in Figs. 5 and 6. The conventional psychrometric chart is extended to accommodate air that is in the supersaturated state. The enthalpy of supersaturated air on the psychrometric chart is given by Eq. (29).

Fig. 5 shows the heating path of the state of the air in a wet-cooling tower for relatively cold inlet air which is saturated with water vapor in this particular example. The path of the air according to the Merkel method is shown as a broken straight line in Fig. 5. The line for the Merkel method is presented as a broken line because straight lines can only be used on psychrometric charts if the temperature of the water surface is constant. The line according to the Merkel method is presented as a straight line because no other information is given by the Merkel method about the humidity of the air except that it is saturated at the air outlet side. That is why the air at the outlet of the cooling tower is assumed to be on the saturation line as shown in Fig. 5.

The Poppe method, on the other hand, gives the state of the air for the entire evaporative process. Since the

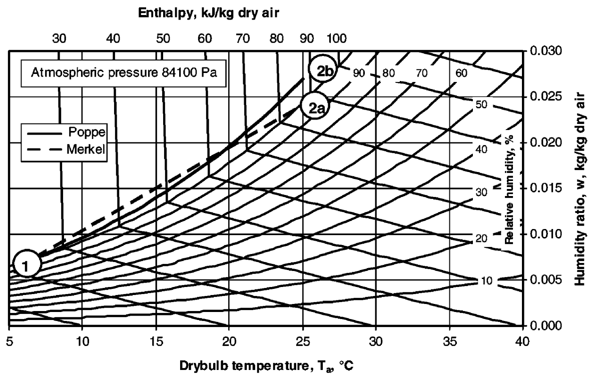

Fig. 5. Path of air in a wet-cooling tower indicated on a supersaturated psychrometric chart.

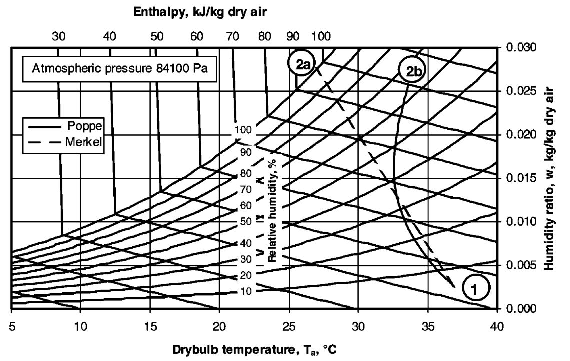

Fig. 6. Psychrometric chart of a water cooling process when the inlet ambient air is hot and very dry.

inlet air is saturated with water vapor, indicated by point 1 in Fig. 5, it immediately becomes supersaturated, according to the Poppe method, as it enters the fill. As the air is heated and the humidity ratio increases, due to the latent heat transfer from the water, it follows the saturation curve very closely. This is because as the air is heated, it can contain more water vapor before it reaches the point of saturation.

Point 2b in Fig. 5 shows the state of the air at the outlet of the heat and mass transfer area or fill in the cooling tower according to the Poppe method. Point 2a in Fig. 5 shows the outlet air state according to the Merkel method. It shows that the air is saturated at the outlet according to Merkel. The outlet air temperatures according to the Merkel and Poppe analyses are relatively close to each other in Fig. 5. The assumption of Merkel that the outlet air is saturated is, therefore, a very good assumption if the actual outlet air temperature is supersaturated.

The degree of supersaturation does not have a great influence on the relative difference between the outlet air temperatures predicted by the Merkel and Poppe analyses. This is because, as seen in Fig. 5, the lines of constant air enthalpy in the supersaturated region are very close to vertical. Therefore, it does not matter how much water vapor and mist are present in the supersaturated air, for a specific air enthalpy, the air temperature will be approximately constant. The difference in the air temperatures at point 2a and 2b in Fig. 5, for the Merkel and Poppe methods, respectively, can be reduced by improving the energy balance employed by the Merkel method where the approximate loss of water due to evaporation in the energy balance is neglected. Refer to Kloppers and Kröger [26] where the loss of water due to evaporation is accounted for in the energy balance.

Fig. 6 shows the heating path of the air in the cooling tower for hot inlet air which is virtually void of water vapor. Point 1 in Fig. 6 shows the state of the inlet air on a psychrometric chart. Point 2b in Fig. 6 shows the state of the air at the outlet of the heat and mass transfer region of the cooling tower according to the Poppe method. The outlet air is colder than the inlet air. Point 2a shows the outlet air state according to the Merkel method.

Fig. 6 shows that the outlet air is saturated according to the Merkel method. The outlet air temperatures, according to the Merkel and Poppe analyses, are not very close to each other. The outlet air temperatures predicted by the Merkel and Poppe analyses lie approximately on the same constant enthalpy line in Fig. 6 as was the case in Fig. 5 when the outlet air was supersaturated according to the Poppe method. In the unsaturated region, however, the lines of constant enthalpy are far from vertical and therefore the large discrepancy in the temperatures. The assumption of Merkel that the outlet air is saturated with water vapor is not as accurate if the actual outlet air is unsaturated as when it is supersaturated.

Fig. 4 shows the enthalpy diagram for the particular example according to the Merkel method while Fig. 7 shows the differences in the enthalpy diagrams between

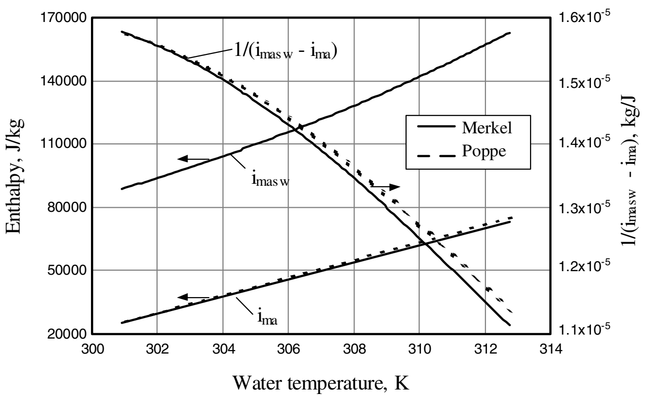

Fig. 7. Enthalpy diagram of the Merkel and Poppe methods.

the Merkel and Poppe methods. The i_{masw} curves of the two methods fall on top of each other. There is a small discrepancy in the i_{ma} curves of the two different methods, especially at the hot water side. It can be seen that the Poppe method predicts an approximately linear variation of the air enthalpy for this specific case but the gradient is different from that predicted by the Merkel method. The $1/(i_{masw} - i_{ma})$ curve of the Poppe method lies above the $1/(i_{masw} - i_{ma})$ curve of the Merkel method. As the transfer characteristic, or Merkel number, is a function of the area under the $1/(i_{masw} - i_{ma})$ curve, the Merkel number according to the Poppe method will be greater than the Merkel number predicted by the Merkel method. It is therefore very important that the same method of method (i.e. Merkel, Poppe or *e-NTU*) be employed in the fill performance test and the subsequent cooling tower performance method.

The results of the Merkel, Poppe and *e-NTU* analyses presented in this study, applied to the evaluation of cooling tower performance, are given in Kloppers and Kröger [26].

6. Conclusion

The governing equations according to the Poppe, Merkel and *e-NTU* methods of method are derived and presented. The governing equations of the Poppe method are expanded to give a more accurate representation of the calculation of the Merkel number. A detailed procedure is presented of how to solve the governing equations with its unique requirements. The Poppe method is especially suited to be employed in the analysis of hybrid cooling towers as the state of the outlet air is accurately determined. The differences between the Merkel and Poppe methods of analysis of evaporative cooling are explained by enthalpy diagrams and psychrometric charts which leads to a better understanding of the implications of the assumptions made by Merkel. The Merkel and *e-NTU* methods of analysis give approximately identical results as it is based on the same simplifying assumptions. It is clear from the discussion that the same method of analysis must be employed in the fill performance test and the subsequent cooling tower performance analysis. The psychrometric chart is expanded to accommodate air that is in the supersaturated state.

Appendix A. Thermophysical properties

The thermophysical properties summarized here are presented in Kröger [7]. Refer to Kröger [7] for the ranges of applicability of the following equations of the thermophysical properties. All the temperatures are expressed in Kelvin.

The enthalpy of the air–water vapor mixture is given by

$$i_{ma} = c_{pa}(T - 273.15) + w[i_{fgwo} + c_{pv}(T - 273.15)] \text{ J/kg dry air} \tag{A.1}$$

where the specific heats, c_{pa} and c_{pv}, are evaluated at $(T + 273.15)/2$ by Eqs. (A.2) and (A.4) respectively. The latent heat i_{fgwo}, is evaluated at 273.15 K according to Eq. (A.8).

The specific heat of dry air is given by

$$c_{pa} = 1.045356 \times 10^3 - 3.161783 \times 10^{-1}T + 7.083814 \times 10^{-4}T^2 - 2.705209 \times 10^{-7}T^3 \text{ J/kgK} \tag{A.2}$$

The vapor pressure of saturated water vapor is given by

$$p_v = 10^z \text{ N/m}^2 \tag{A.3}$$

where

$$z = 10.79586(1 - 273.16/T) + 5.02808\log_{10}(273.16/T) + 1.50474 \times 10^{-4}[1 - 10^{-8.29692\{(T/273.16)-1\}}] + 4.2873 \times 10^{-4}[10^{4.76955(1-273.16/T)} - 1] + 2.786118312$$

The specific heat of saturated water vapor is given by

$$c_{pv} = 1.3605 \times 10^3 + 2.31334T - 2.46784 \times 10^{-10}T^5 + 5.91332 \times 10^{-13}T^6 \text{ J/kgK} \tag{A.4}$$

The specific heat of mixtures of air and water vapor is given by

$$c_{pma} = (c_{pa} + wc_{pv}) \text{ J/K kg dry air} \tag{A.5}$$

The humidity ratio is given by

$$w = \left(\frac{2501.6 - 2.3263(T_{wb} - 273.15)}{2501.6 + 1.8577(T - 273.15) - 4.184(T_{wb} - 273.15)}\right)\left(\frac{0.62509p_{vwb}}{p_a - 1.005p_{vwb}}\right) \times \left(-\frac{1.00416(T - T_{wb})}{2501.6 + 1.8577(T - 273.15) - 4.184(T_{wb} - 273.15)}\right) \tag{A.6}$$

where p_{vwb} is the vapor pressure from Eq. (A.3) evaluated at the wetbulb temperature.

The specific heat of water is given by

$$c_{pw} = 8.15599 \times 10^3 - 2.80627 \times 10T + 5.11283 \times 10^{-2}T^2 - 2.17582 \times 10^{-13}T^6 \text{ J/kgK} \tag{A.7}$$

The latent heat of water is given by

$$i_{fgw} = 3.4831814 \times 10^6 - 5.8627703 \times 10^3T + 12.139568T^2 - 1.40290431 \times 10^{-2}T^3 \text{ J/K} \tag{A.8}$$

i_{fgwo} is obtained from Eq. (A.8) where $T = 273.15$.

Appendix B. Solving the system of differential equations

B.1. Solving procedure

The fourth order Runge–Kutta method [5,22,23] is employed to solve the system of differential equations for unsaturated and supersaturated air. The system of equations for unsaturated air (including saturated air) is represented by Eqs. (20), (21) and (26). The system of equations for supersaturated air is represented by Eqs. (38), (39) and (43). In the equations that follow, i_{ma} must be replaced by i_{ss} for supersaturated air.

The first step in the solution process is to divide the fill into a number of intervals where the water temperature difference is equal across each interval, i.e.

$$\Delta T_w = (T_{wi} - T_{wo})/(\text{Number of intervals}) \tag{B.1}$$

Fig. B.1 shows an example where the fill is divided into five intervals. It is necessary to divide the fill into more than one interval to capture, as accurately as possible, the point at which the air becomes supersaturated. This is because a different set of equations is applicable for supersaturated air. Approximately five intervals are generally sufficient to obtain accurate results. It was mentioned that the value of w_o is not known a priori. A value of w_o is guessed and a new value of w_o is subsequently determined. The equations are solved until the value of w_o converges. Only a few of these iterations are generally necessary to obtain convergence.

The equations are solved across one interval at a time by the Runge–Kutta method, which is explained in Section B.2. The air, which is generally unsaturated, enters the fill at Level (0) in Fig. B.1 with w_i, i_{mai}, m_a known. The values of $w_{(1)}$ and $i_{ma(1)}$ are then determined by the Runge–Kutta method with the set of equations for unsaturated air. m_a remains constant. It is then determined whether the air is still unsaturated or if it is supersaturated at the outlet of the first interval (i.e. at level (1) in Fig. B.1). If the air is supersaturated, the set of equations for supersaturated air must be solved across the next interval. If the air is supersaturated it will generally

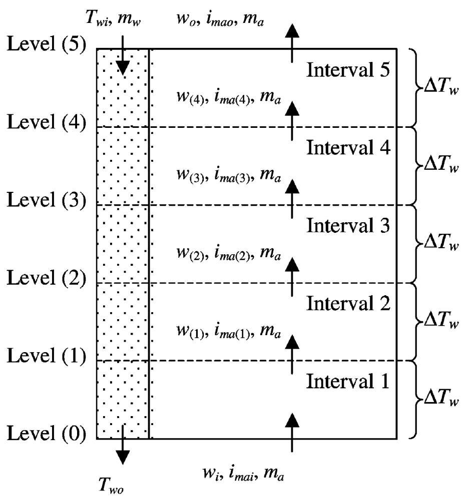

Fig. B.1. Counterflow fill divided into five intervals.

remain in the supersaturated state through the rest of the fill.

The following procedure can be followed to determine whether the air at the outlet of an interval, as indicated in Fig. B.1, is unsaturated or supersaturated: assume that the air at level (1), for example, is unsaturated and determine $T_{a(1)}$ from Eq. (A.1) by iterative means with $w_{(1)}$ and $i_{ma(1)}$ known. Then assume that the air is saturated and determine the wetbulb temperature, $T_{wb(1)}$ from Eq. (A.6). $T = T_{wb}$ in Eq. (A.6) when the air is saturated. If $T_{a(1)} > T_{wb(1)}$ then the assumption that the air is unsaturated is correct. If $T_{wb(1)} > T_{a(1)}$, which is impossible, the air is supersaturated. The actual value of the wetbulb temperature is then $T_{wb(1)} = T_{a(1)}$.

B.2. Implementation of the fourth order Runge–Kutta method

Eqs. (20), (21) and (26) for unsaturated and saturated air or Eqs. (38), (39) and (43) for supersaturated air can be respectively written as

$$\frac{dw}{dT_w} = f(w, i_{ma}, T_w) \tag{B.2}$$

$$\frac{di_{ma}}{dT_w} = g(w, i_{ma}, T_w) \tag{B.3}$$

$$\frac{dMe_P}{dT_w} = h(w, i_{ma}, T_w) \tag{B.4}$$

Refer to Fig. B.1. The cooling tower fill is divided into one or more intervals with the same water temperature difference across each interval. In addition to the intervals, levels are specified (a level is an imaginary horizontal plane through the fill at the top and bottom of the fill and between two fill intervals). Initial values of the variables w, i_{ma} and T_w are required on a particular level, say level (n). The values of the variables can then be determined at level ($n+1$) with the aid of Eqs. (B.5)–(B.7).

$$w_{(n+1)} = w_{(n)} + (j_{(n+1,1)} + 2j_{(n+1,2)} + 2j_{(n+1,3)} + j_{(n+1,4)})/6 \tag{B.5}$$

$$i_{ma(n+1)} = i_{ma(n)} + (k_{(n+1,1)} + 2k_{(n+1,2)} + 2k_{(n+1,3)} + k_{(n+1,4)})/6 \tag{B.6}$$

$$Me_{P(n+1)} = Me_{P(n)} + (l_{(n+1,1)} + 2l_{(n+1,2)} + 2l_{(n+1,3)} + l_{(n+1,4)})/6 \tag{B.7}$$

where

$$j_{(n+1,1)} = \Delta T_w \cdot f(T_{w(n)}, i_{ma(n)}, w_{(n)}) \tag{B.8}$$

$$k_{(n+1,1)} = \Delta T_w \cdot g(T_{w(n)}, i_{ma(n)}, w_{(n)}) \tag{B.9}$$

$$l_{(n+1,1)} = \Delta T_w \cdot h(T_{w(n)}, i_{ma(n)}, w_{(n)}) \tag{B.10}$$

$$j_{(n+1,2)} = \Delta T_w \cdot f\left(T_{w(n)} + \frac{\Delta T_w}{2}, i_{ma(n)} + \frac{k_{(n+1,1)}}{2}, w_{(n)} + \frac{j_{(n+1,1)}}{2}\right) \tag{B.11}$$

$$k_{(n+1,2)} = \Delta T_w \cdot g\left(T_{w(n)} + \frac{\Delta T_w}{2}, i_{ma(n)} + \frac{k_{(n+1,1)}}{2}, w_{(n)} + \frac{j_{(n+1,1)}}{2}\right) \tag{B.12}$$

$$l_{(n+1,2)} = \Delta T_w \cdot h\left(T_{w(n)} + \frac{\Delta T_w}{2}, i_{ma(n)} + \frac{k_{(n+1,1)}}{2}, w_{(n)} + \frac{j_{(n+1,1)}}{2}\right) \tag{B.13}$$

$$j_{(n+1,3)} = \Delta T_w \cdot f\left(T_{w(n)} + \frac{\Delta T_w}{2}, i_{ma(n)} + \frac{k_{(n+1,2)}}{2}, w_{(n)} + \frac{j_{(n+1,2)}}{2}\right) \tag{B.14}$$

$$k_{(n+1,3)} = \Delta T_w \cdot g\left(T_{w(n)} + \frac{\Delta T_w}{2}, i_{ma(n)} + \frac{k_{(n+1,2)}}{2}, w_{(n)} + \frac{j_{(n+1,2)}}{2}\right) \tag{B.15}$$

$$l_{(n+1,3)} = \Delta T_w \cdot h\left(T_{w(n)} + \frac{\Delta T_w}{2}, i_{ma(n)} + \frac{k_{(n+1,2)}}{2}, w_{(n)} + \frac{j_{(n+1,2)}}{2}\right) \tag{B.16}$$

$$j_{(n+1,4)} = \Delta T_w \cdot f\left(T_{w(n)} + \Delta T_w, i_{ma(n)} + k_{(n+1,3)}, w_{(n)} + j_{(n+1,3)}\right) \tag{B.17}$$

$$k_{(n+1,4)} = \Delta T_w \cdot g\left(T_{w(n)} + \Delta T_w, i_{ma(n)} + k_{(n+1,3)}, w_{(n)} + j_{(n+1,3)}\right) \tag{B.18}$$

$$l_{(n+1,4)} = \Delta T_w \cdot h\left(T_{w(n)} + \Delta T_w, i_{ma(n)} + k_{(n+1,3)}, w_{(n)} + j_{(n+1,3)}\right) \tag{B.19}$$

The four variables in the Runge–Kutta method are T_w, w, i_{ss} or i_{ma} and Me_P from the left-hand side of Eqs. (20), (21) and (26) for unsaturated air and Eqs. (38), (39) and (43) for supersaturated air. For this reason Eqs. (B.2)–(B.4) are functions of only w, i_{ma} or i_{ss} and

T_w. Most of the other variables are functions of these variables. Eqs. (B.2)–(B.4) are not functions of Me_P because dMe_P/dT_w is a function of dw/dT_w as can be seen from Eq. (42). Thus, Eqs. (20) and (21) for unsaturated air, or Eqs. (38) and (39) for supersaturated air can be solved without Eq. (26) or Eq. (43) respectively.

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